Simulation of heap leaching process in a bed of porous and spherical ore particles

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Abstract

Purpose – The purpose of this paper is to apply the numerical methods to study the heap leaching process in a bed of porous and spherical ore particles. This study is performed in two stages: first, modeling the leaching process of a soluble mineral from a spherical and porous ore particle to obtain the distribution of mineral concentrations, leaching solvent concentration and dissolved mineral in the particles (the particle model), and second, modeling the heap leaching of the mineral from a porous bed consisting of spherical and porous ore particles to obtain the distribution of mineral concentrations, leaching solvent concentration and dissolved mineral in the bed (the bed model).

Design/methodology/approach – The governing equations are derived for the particle model, and then converted into non-dimensional form using reference quantities. The non-dimensional equations are discretised on a uniform spherical grid fitted to the particle using finite difference method. The resulting algebraic equations are solved using Tri-Diagonal Matrix Algorithm, and the governing equations are derived for the bed model, and then converted into non-dimensional form using reference quantities. The non-dimensional equations are discretised explicitly on a one-dimensional and uniform grid fitted to the bed. The unknown quantities are evaluated using the corresponding values at the previous time interval.

Findings – The results obtained from numerical modeling show that, when the particle has a low diffusion resistance or a high chemical resistance, the reaction takes place slowly and homogeneously throughout the ore particle. On the other hand, when the bed has a low convection resistance, the reaction takes place homogeneously throughout the bed. As the convection resistance increases, the non-homogeneous (local) behavior predominates. It is also noticed that, when the chemical reaction resistance is high, the reaction takes place homogeneously throughout the bed.

Research limitations/implications – The dynamic diffusion and movement of solution in the ore particles and ore bed are not modeled and volumetric ratio of solution in the particles and the bed and also vertical velocity of solution in the bed are assumed to be fixed constants.

Practical implications – This study shows that the reaction takes place homogeneously throughout the bed when the convection resistance is low, the diffusion resistance is high, the concentration resistance is low, and the chemical reaction resistance is high.

Originality/value – Homogeneous reaction conditions being suitable for heap leaching operations are identified. Thus, it is recommended to approach the above conditions when preparing ore heaps and designing the relevant operation.

Keywords Simulation, Convection, Chemical reactions, Solvents

Paper type Research paper

Nomenclature

A	= Leaching solvent	α	= Dimensionless parameter (equation (21))	
b	= Stoichiometric coefficient			
B	= Mineral (equation (1))	β	= Dimensionless parameter (equation (21))	
B	= Bed			
C	= Dissolved mineral (equation (1))	δ	= Dimensionless parameter (equation (27))	
C	= Concentration (equation (2))			
C	= Conversion (equation (28))	ε	= Porosity	
D	= Diffusion coefficient (equation (2))	ϕ	= Reaction order	
D	= Diffusion (equation (29))	κ	= Dimensionless parameter (equation (27))	
E	= Extraction			
eff	= Effective	λ	= Dimensionless parameter (equation (27))	
F	= Fraction			
H	= Height of the bed	ν	= Dimensionless parameter (equation (27))	
k	= Reaction rate constant			
l	= Solution	θ	= Dimensionless parameter (equation (21))	
p	= Particle			
r	= Radial direction	ρ	= Density	
R	= Ore radius	σ	= Dimensionless parameter (equation (21))	
R	= Reaction (Figure 8)			
S	= Source/sink term	τ	= Tortosity	
SCC	= Sarcheshmeh Copper Complex	ω	= Dimensionless parameter (equation (27))	
TMS	= The Minerals, Metals and Materials Society	ξ	= Dimensionless parameter (equation (21))	
t	= Time	ζ	= Dimensionless parameter (equation (21))	
u	= Velocity			
z	= Depth of the bed			

1. Introduction

Nowadays, some of the copper supply in the world is obtained by hydrometallurgical methods. Among these methods, heap leaching is used for metal extraction from secondary and oxide ores. In recent years, heap leaching has been the subject of many researches carried out in the metal extraction area. Mathematical modeling of this operation provides useful sights for design purposes. Thus, many researchers have attempted to study and simulate the important phenomena in this process.

Most of general models in spherical coordinates, describing the kinetics of leaching in low-grade ores, have been presented by Bartlett (1973). The results of column testing of oxide ores using unreacted core model are simulated by Roman *et al.* (1974). They also presented a model for industrial leaching of copper oxide ores. This model is for one dimensional and isothermal state. He assumed pseudo-steady-state kinetics and used unreacted core model for particles. Shafer *et al.* (1979) simulated the experimental results of leaching oxide ores using unreacted core model. Chae and Wadsworth (1989) applied the reaction zone model for simulation of *in situ* leaching of copper oxide ores. This model is for one-dimensional and isothermal situation and assumes pseudo-steady-state kinetics with unreacted core model for particle. Bartlett (1992a) described the solution mining process in details. He studied the ore bed space and distribution of the void space between the particles that are distributed between the solution and air phases. Bartlett (1992b) reviewed the models that have been used for heap leaching. According to the available literature and the published results, he

presented several deterministic models that are used in different conditions. Dixon and Hendrix (1993a, b) have presented an unsteady and general model for leaching one or more reactant from a non-reactive porous and spherical pellet and from a bed consisting of such pellets. This dimensionless model was used for interpreting the leaching data, based on which the design of industrial leaching processes can be accomplished. In this work chemical reaction rate, diffusion rate, and competition of reactants for reaction and the importance of reactant fraction on the ore surface have been studied. Bartlet (1997) in his lecture at TMS described the details of heap leaching process and the effective mechanism of ore leaching. He also presented the results of available simulation and suggested the strategies for optimization of this operation. The processes of acid diffusion, reaction with solid reactant and diffusion of dissolved specie in a spherical and porous ore particle and also extraction of one solid reactant from an ore bed are simulated by Sheikhzadeh *et al.* (2002a, b).

The purpose of this paper is to simulate the extraction of one leachable mineral from spherical and porous ore particles and heap leaching of the mineral from an ore bed consisting of such particles using mathematical and computational methods. In order to achieve this goal, it is assumed that the bed consists of uniform, spherical and porous ore particles. The governing equations and initial and boundary conditions for the diffusion of leaching agent in the particles, reaction of agent with mineral and diffusion of dissolved mineral towards the ore surface and convection of agent and dissolved mineral in the bed are derived and converted to dimensionless form. Finally, these dimensionless equations are solved using a suitable numerical method and the results are discussed.

In this paper, a parametric study has been performed based on two parameters ν and λ experimentally measured for heap leaching operation in SCC. The dynamic diffusion and movement of solution in the ore particles and ore bed are not modeled and volumetric ratio of solution in the particles and the bed and also vertical velocity of solution in the bed are assumed fixed constants. The model has been calibrated using experimental results obtained at SCC (Sheikhzadeh *et al.*, 2005). The applicability of the model has been verified when the numerical predictions are in good agreement with the test results (Sheikhzadeh, 2004).

2. Governing equations

In order to simulate processes in the heap leaching, two models are used, namely particle model and bed model. The particle model concerns the leaching of mineral from an ore particle and the bed model concerns the leaching of mineral from the ore bed. These two models are coupled through the mass transfer at the interface of particles and the bed (surface of particles).

2.1 Particle model

The porous and spherical ore particle with uniform structure is shown in Figure 1. Leachable mineral is uniformly distributed in pore walls of this ore particle. It is assumed that the dilute leaching solution flows symmetrically among the particles, thus the properties of the solution at the surface of particles are determined according to the solution in the bed.

Parametric equation of reaction for mineral deposits with leaching solvent available in the pore solution is:

$$A + bB \rightarrow C \qquad (1)$$

Owing to solvent consumption in the pore solution, solvent concentration from the ore surface to the ore center has a non-zero gradient and due to dissolving the mineral in the pore solution, the dissolved mineral concentration from the ore center to the ore surface has also a non-zero gradient.

The mass balance of the leaching solvent on a spherical differential element of the particle requires the increasing (or decreasing) rate of the solvent inside the element equals the net mass flow rate into it and production (or consumption) rate. The continuity equation (in terms of leaching solvent mole per unit volume of the ore per second) is then obtained as:

$$\varepsilon_p \frac{\partial C_A}{\partial t} = \frac{D_{A\text{eff}}}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial C_A}{\partial r}\right) + S_{C_A} \qquad (2)$$

The effective diffusion coefficient is defined:

$$D_{A\text{eff}} = \frac{D_A \varepsilon_p}{\tau} \qquad (3)$$

Tortosity, τ, is the effect of movement of dissolved species in an indirect path through the pore structure rather than a stochastic movement without any barrier along the chemical potential gradient or concentration gradient. However, the effective diffusion coefficient must be determined experimentally. Owing to chemical reaction between the solvent and mineral, some solvent is consumed and the source term in equation (2) is negative and is determined if the chemical reaction rate is known.

In a similar manner, the continuity equation for diffusion of dissolved reaction products through the pore solution toward the ore surface is:

$$\varepsilon_p \frac{\partial C_C}{\partial t} = \frac{D_{C\text{eff}}}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial C_C}{\partial r}\right) + S_{C_C} \qquad (4)$$

It is assumed that the dissolution rate of the mineral in pore solution is related to the mineral concentration in the ore and solvent concentration:

$$\frac{dC_B}{dt} = -kC_B^\phi C_A \qquad (5)$$

This equation can be applied at any ore radius.
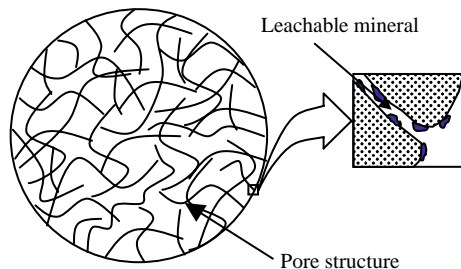
Figure 1.
A spherical porous ore
particle

According to equation (1) for dissolving N_B moles of mineral, N_B/b moles of leaching solvent are consumed. Therefore, the source terms S_{C_A} and S_{C_C} from equations (1) to (5) become:

$$S_{C_A} = -\rho(1 - \varepsilon_p)\frac{kC_B^\phi C_A}{b}, \quad : S_{C_C} = \rho(1 - \varepsilon_p)kC_B^\phi C_A \tag{6}$$

To obtain the distribution of leaching solvent concentration, mineral concentration and dissolved reactant concentration in solution within the ore at any time interval, equations (2), (4) and (5) with the source terms defined in equation (6) should be solved simultaneously. In order to solve these equations the initial and boundary conditions must be defined.

One initial condition and two boundary conditions are required for solving the continuity equation of leaching solvent. At the start of process, initial concentration of leaching solvent is zero throughout the ore. Therefore, the initial condition for equation (2) becomes:

$$C_A(r, z, 0) = 0 \tag{7}$$

Leaching solvent concentration on the ore surface at any time is determined according to leaching solvent concentration in the bed, therefore the boundary conditions of equation (2) is:

$$C_A(R, z, t) = C_{AB}(z, t) \tag{8}$$

Another boundary condition for equation (2) is specified from symmetry as follows:

$$\left.\frac{\partial C_A}{\partial r}(r, z, t)\right|_{r=0} = 0 \tag{9}$$

equation (4) requires one initial condition and two boundary conditions as follows:

$$C_C(r, z, 0) = 0 \tag{10}$$

$$C_C(R, z, t) = C_{CB}(z, t) \tag{11}$$

$$\left.\frac{\partial C_C}{\partial r}(r, z, t)\right|_{r=0} = 0 \tag{12}$$

Equation (5) requires one initial condition. Initial concentration of mineral in the ore is known, therefore:

$$C_B(r, z, 0) = C_{B0} \tag{13}$$

2.2 Bed model

The ore bed consisting of porous and spherical particles with uniform structure is shown in Figure 2. It is assumed that the average radius of particles is R and solution flows among the particles with constant velocity u_s. Solution flow through the bed and mass transfer between the bulk solution and particle solution are assumed ideal.

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Figure 2.
Schematic diagram of the
ore bed
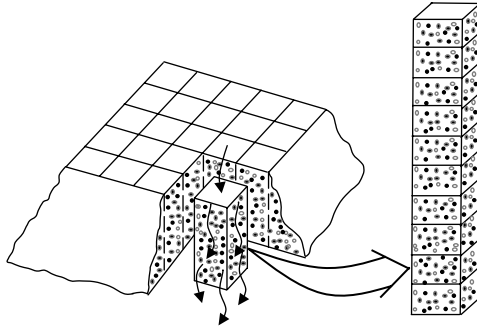
The mass balance of the leaching solvent on a one dimensional differential element of the bed and neglecting the diffusion term in comparison with the convection term, gives the continuity equation of leaching solvent as:

$$\varepsilon_l \frac{\partial C_{AB}}{\partial t} + u_s \frac{\partial C_{AB}}{\partial z} = S_{C_{AB}} \tag{14}$$

In a similar manner, the continuity equation for dissolved mineral inside the bed solution, C_{CB}, is obtained as:

$$\varepsilon_l \frac{\partial C_{CB}}{\partial t} + u_s \frac{\partial C_{CB}}{\partial z} = S_{C_{CB}} \tag{15}$$

Based on the mass transfer on the surface of the particles, the sink/source terms $S_{C_{AB}}$ and $S_{C_{CB}}$ become:

$$S_{C_{AB}} = -\frac{3(1-\varepsilon_B)}{R} D_{A\,\text{eff}} \frac{\partial C_A}{\partial r}\bigg|_{r=R}, \quad S_{C_{CB}} = -\frac{3(1-\varepsilon_B)}{R} D_{C\,\text{eff}} \frac{\partial C_C}{\partial r}\bigg|_{r=R} \tag{16}$$

To obtain the distribution of leaching solvent concentration and dissolved mineral concentration in the solution within the bed at any time interval, equations (14) and (15) with the source terms defined in equation (16) together with the particle model equations should be solved simultaneously. In order to solve these equations the initial and boundary conditions must be defined.

One initial condition and one boundary condition are required for solving the continuity equation of leaching solvent and dissolved mineral. At the start of process, initial concentration of leaching solvent is zero throughout the bed. Therefore, the initial condition for equation (14) becomes:

$$C_{AB}(z, 0) = 0 \tag{17}$$

Leaching solvent concentration on the top surface of bed is equal to leaching solvent concentration of solution infiltered to the bed, C_{A0}, thus the boundary condition of equation (14) is:

$$C_{AB}(0, t) = C_{A0} \tag{18}$$

At the start of process, initial concentration of dissolved mineral is zero throughout the bed. Therefore, the initial condition for equation (15) becomes:

$$C_{CB}(z, 0) = 0 \tag{19}$$

Concentration of dissolved mineral on the top surface of bed is zero, therefore the boundary condition of equation (15) is:

$$C_{CB}(0, t) = 0 \tag{20}$$

3. Governing equations in dimensionless form

The variables in the governing equations and their initial and boundary conditions are relatively numerous, therefore to reduce variables and specify the important parameters, it is convenient to convert these equations to dimensionless form. Having the dimensionless results we can carry out parametric study and investigate the effect of important parameters on extraction of mineral from the ore particle and the ore bed. For this purpose we use the following dimensionless variables:

$$\alpha = \frac{C_A}{C_{A0}} \quad \alpha_B = \frac{C_{AB}}{C_{A0}} \quad \sigma = \frac{C_C}{bC_{A0}} \quad \sigma_B = \frac{C_{CB}}{bC_{A0}}$$
$$\beta = \frac{C_B}{C_{B0}} \quad \zeta = \frac{z}{H} \quad \xi = \frac{r}{R} \quad \theta = \frac{u_s t}{\varepsilon_l H} \tag{21}$$

Substituting dimensionless variables, the continuity equations for leaching solvent, dissolved reactant, reaction rate and their initial and boundary conditions of particle model become:

$$\frac{3}{\nu\omega} \frac{\partial \alpha}{\partial \theta} = \frac{1}{\xi^2} \frac{\partial}{\partial \xi} \left(\xi^2 \frac{\partial \alpha}{\partial \xi} \right) - \kappa \beta^\phi \alpha,$$

$$\alpha(\xi, \zeta, 0) = 0, \quad \alpha(1, \zeta, \theta) = \alpha_B(\zeta, \theta), \quad \left. \frac{\partial \alpha}{\partial \xi}(\xi, \zeta, \theta) \right|_{\xi=0} = 0 \tag{22}$$

$$\frac{3}{\nu\omega} \frac{\partial \sigma}{\partial \theta} = \frac{\delta}{\xi^2} \frac{\partial}{\partial \xi} \left(\xi^2 \frac{\partial \sigma}{\partial \xi} \right) + \kappa \beta^\phi \alpha,$$

$$\sigma(\xi, \zeta, 0) = 0, \quad \sigma(1, \zeta, \theta) = \sigma_B(\zeta, \theta), \quad \left. \frac{\partial \sigma}{\partial \xi}(\xi, \zeta, \theta) \right|_{\xi=0} = 0 \tag{23}$$

$$\frac{d\beta}{d\theta} = -\frac{\kappa \lambda \nu \omega}{3} \beta^\phi \alpha, \quad \beta(\xi, \zeta, 0) = 1, \tag{24}$$

The continuity equations for leaching solvent and dissolved reactant and their initial and boundary conditions of bed model become:

$$\frac{\partial \alpha_B}{\partial \theta} + \frac{\partial \alpha_B}{\partial \zeta} = -\omega \left. \frac{\partial \alpha}{\partial \xi} \right|_{\xi=1}, \quad \alpha_B(\zeta, 0) = 0, \quad \alpha_B(0, \theta) = 1 \tag{25}$$

$$\frac{\partial \sigma_B}{\partial \theta} + \frac{\partial \sigma_B}{\partial \zeta} = -\delta\omega\frac{\partial \sigma}{\partial \xi}\bigg|_{\xi=1}, \quad \sigma_B(\zeta, 0) = 0, \quad \sigma_B(0, \theta) = 0 \tag{26}$$

Dimensionless parameters in these equations are:

$$\kappa = \frac{\rho(1 - \varepsilon_P)kC_{B0}^{\phi}R^2}{bD_{A\text{eff}}} \quad \lambda = \frac{\varepsilon_P bC_{A0}}{\rho(1 - \varepsilon_P)C_{B0}} \quad \delta = \frac{D_{C\text{eff}}}{D_{A\text{eff}}}$$

$$\nu = \frac{\varepsilon_l}{\varepsilon_P(1 - \varepsilon_B)} \quad \omega = \frac{3(1 - \varepsilon_B)D_{A\text{eff}}H}{u_s R^2} \tag{27}$$

It should be noted that θ is the dimensionless time of convection in the bed, $\nu\omega\theta/3 = \theta_D$ is the dimensionless time of diffusion in the particle, and $\kappa\lambda\theta_D = \theta_R$ is dimensionless time of reaction in the particle.

4. Fraction of conversion and fraction of extraction
Solving the above equations, gives the distribution of concentrations of leaching solvent in the ore particles and bed, mineral in the ore particles and dissolved reactant in pore solution of ore and bed at any time interval. Changing the important parameters, different cases can be studied. Fraction of conversion and fraction of extraction are defined to describe the overall changes of the system with time.

Fraction of conversion, F_C, is defined as fraction of extractable mineral present in the particle dissolved at any time interval:

$$F_C = \int_0^1 3(1 - \beta)\xi^2 d\xi \tag{28}$$

β is 1 at the start of process and 0 at the end, therefore F_C changes between 0 and 1.

Fraction of extraction, F_E, is defined as fraction of extractable mineral present in the particle that has been dissolved in pore solution and exits from the particle at any time:

$$F_E = \int_0^{\theta_D} -\delta\lambda\left(\frac{\partial \sigma}{\partial \xi}\right)_{\xi=1} d\theta_D \tag{29}$$

5. Numerical procedure
A uniform spherical grid is fitted to the particle and a one dimensional and uniform grid to the bed. The conservation equations are discretized on these grids using the finite difference method. A fully implicit method is used to convert the continuity equation of leaching solvent and dissolved reactant in the particle to algebraic equations. An explicit method is used to convert the continuity equation of leaching solvent and dissolved reactant in the bed to algebraic equations. The discretization method for the space terms is central differencing and for the time terms is forward differencing. The resulting algebraic equations for the particle model are solved using Tri-Diagonal Matrix Algorithm (Patankar, 1980) and computations continue iteratively in each time level to achieve the converged solution. The reaction rate equation for the particle model is solved using an iterative method. The discretization method for the bed model equations being explicit, the unknown quantities are evaluated using the corresponding values at the previous time interval. The fraction of conversion, F_C, and fraction of extraction, F_E, are evaluated using the Simpson integrating rule.

6. Results and discussion

In order to reduce the number of variables, it is assumed that the effective diffusion coefficient of solvent is equal to the effective diffusion coefficient of dissolved reactant, i.e. $\delta = 1$. Therefore, there are four variable parameters (κ, λ, ω, ν) upon their variations, different cases can be achieved.

6.1 The results of particle model

The results of particle model are presented for a particle at the top surface of the bed. At this point of the bed, the leaching solvent concentration of the bed solution is the same as the leaching solvent concentration of the infiltered solution and the dissolved reactant concentration of the bed solution is zero, i.e. $\alpha_B = 1$ and $\sigma_B = 0$.

The obtained results for the case that the reaction rate equation is of the first order with respect to the mineral concentration (σ_B), i.e. $\phi = 1$, is reviewed first. When $\kappa = 1$ and $\lambda = 100$, changes of leaching solvent concentration, mineral and dissolved reactant in the particle at several time intervals are shown in Figure 3.
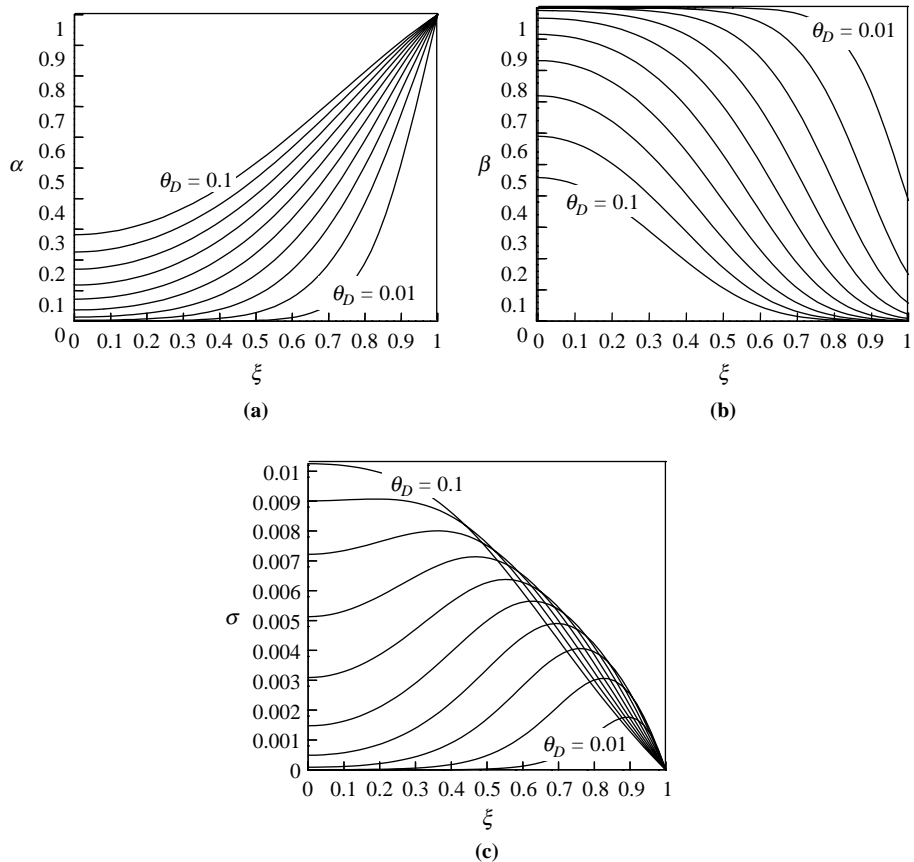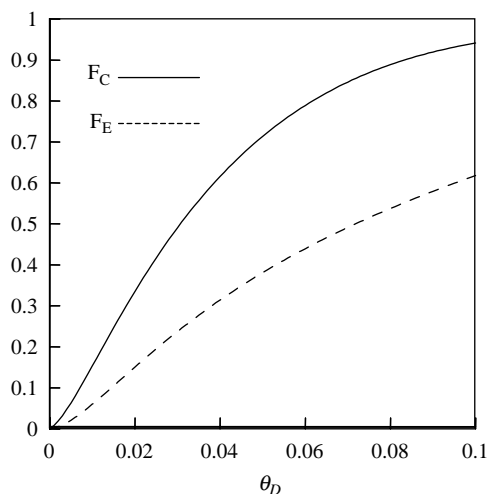

Figure 3.
Concentration of:
(a) leaching solvent,
(b) mineral, (c) dissolved
mineral in terms of radius
of the ore particle for
$\kappa = 1$, $\lambda = 100$ at several
time intervals
($\Delta\theta_D = 0.01$)

Figure 3(a) shows that at the first time intervals, leaching solvent concentration in the interior core of particle is zero and then leaching solvent is diffused from the particle surface towards the particle center and leaching solvent concentration gradient that is initially steep, gradually approaches zero. Figure 3(b) shows that the mineral concentration is reduced with time and when $\theta_D = 0.1$, most of the extractable mineral available in the ore is dissolved. Figure 3(c) shows that at the start of process, concentration of dissolved reactant is zero at the surface and central core of the particle and its maximum value, is occurred at a radius close to the particle surface. As the time goes on, the dissolved reactant concentration is increased and the point corresponding to the maximum value of concentration moves towards the center of particle. It should be noted that, a large amount of dissolved reactant exits from the particle and therefore the concentration of dissolved reactant present in the particle is much lower than the amount of dissolved reactant.

The fraction of conversion and the fraction of extraction are shown in terms of time in Figure 4. It is observed that, as the time goes on, the fraction of conversion approaches unity. At a later time the fraction of extraction approaches unity too. It can be observed that, when dimensionless diffusion time approaches to 0.1, the fraction of conversion and the fraction of extraction are nearly 0.94 and 0.62, respectively.

Changes of leaching solvent concentration and mineral concentration for different combinations of $\lambda = 0.01, 100$ and $\kappa = 0.01, 100$ are shown in Figure 5. It is observed that, in the case of small particle containing a little mineral (low grade ore), $\kappa = 0.01$, homogeneous reaction in the whole particle is occurred and increasing λ does not have considerable effect on homogeneity of reaction. Small κ denotes high diffusion rate or low diffusion resistance. Therefore, if κ is small, homogenous reaction is occurred in the whole particle and there is not a reaction zone. However, in the case that the diffusion resistance approaches zero, it is not required to solve the leaching solvent continuity equation and the reaction rate equation would be enough.
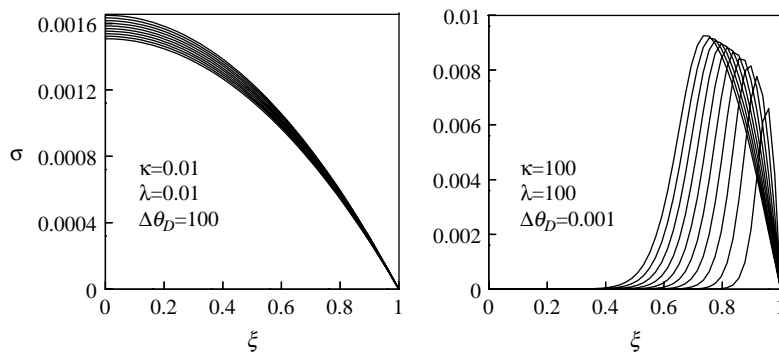

Figure 4.
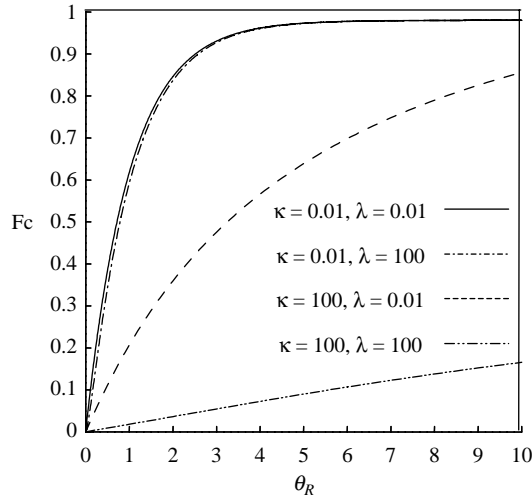Fraction of conversion and fraction of extraction in terms of time for $\kappa = 1$, $\lambda = 100$
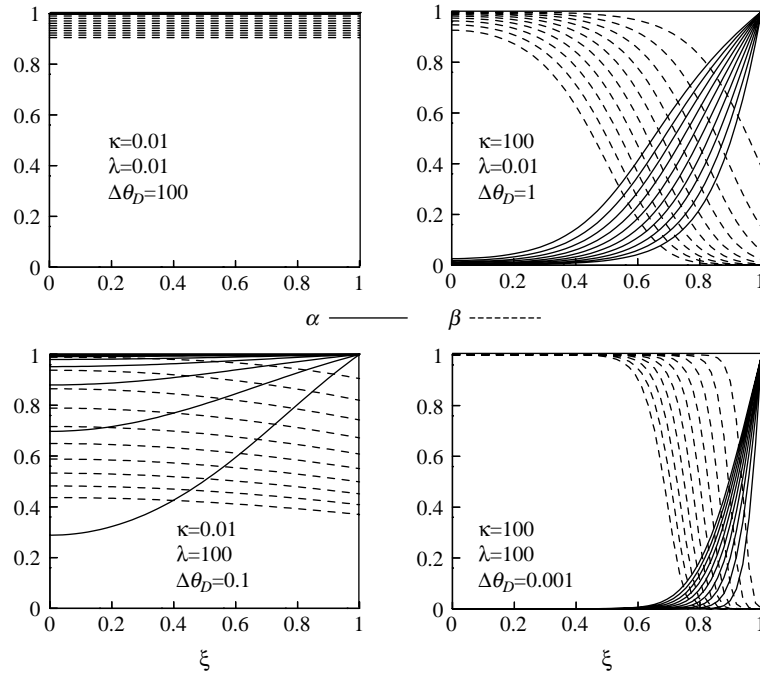
Figure 5.
Comparison of
concentrations of acid and
mineral in the ore particle
for several κ and λ

Changes of dissolved reactant for two cases $\lambda = 0.01$, $\kappa = 0.01$ and $\lambda = 100$, $\kappa = 100$ are shown in Figure 6. It is observed that, in the case that the diffusion resistance is small, $\kappa = 0.01$, the maximum value of dissolved reactant concentration occurs at the center of particle, but in the case that the diffusion resistance is high and there is a reaction zone in the particle, $\kappa = 100$, the maximum value of dissolved reactant concentration occurs in the reaction zone and later approaches to the center of the particle.

Changes of the fractional conversion in terms of dimensionless reaction time, θ_R, for different combinations of $\lambda = 0.01, 100$ and $\kappa = 0.01, 100$, are shown in Figure 7. It can be observed that, when the diffusion resistance is low, $\kappa = 0.01$, fractions of
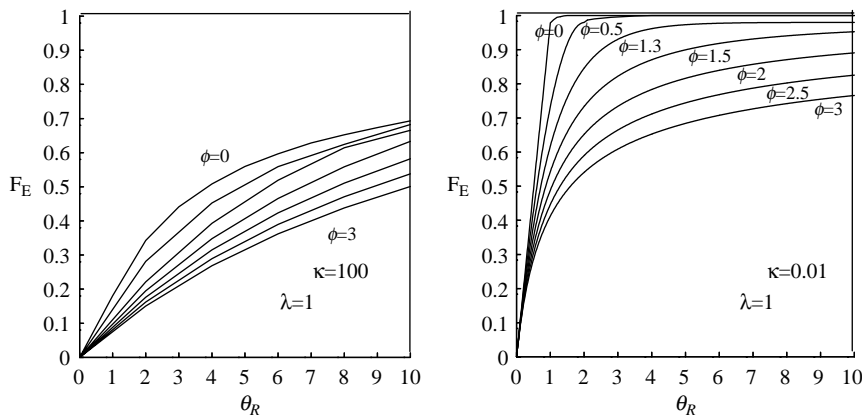

Figure 6.
Comparison of dissolved
mineral concentrations in
the ore particle for two
values of κ and λ

Figure 7.
Variations of fraction of
conversion for two values
of κ and λ

conversion for $\lambda = 0.01, 100$ are the same. In this case, the chemical reaction controls the process and the leaching solvent strength does not have considerable effect on dissolving the mineral, thus the reaction conversion rates are the same. Increasing κ, reduces the relative rate of conversion and increasing λ, enhances this reduction. At a certain θ_R, if κ is constant, and λ increases, the dimensionless diffusion time, θ_D, becomes lower and the required time for diffusion is lower. Thus, increasing λ lowers conversion because of diffusion-controlled reaction.

The effect of ϕ on F_C is shown in Figure 8. It can be observed that if κ is small, changes of ϕ has important effect on F_C. At low κ, the conversion rate is controlled by chemical reaction and the total reaction is affected by chemical kinetics and then the above changes become considerable. At high κ, the diffusion rate is more important than
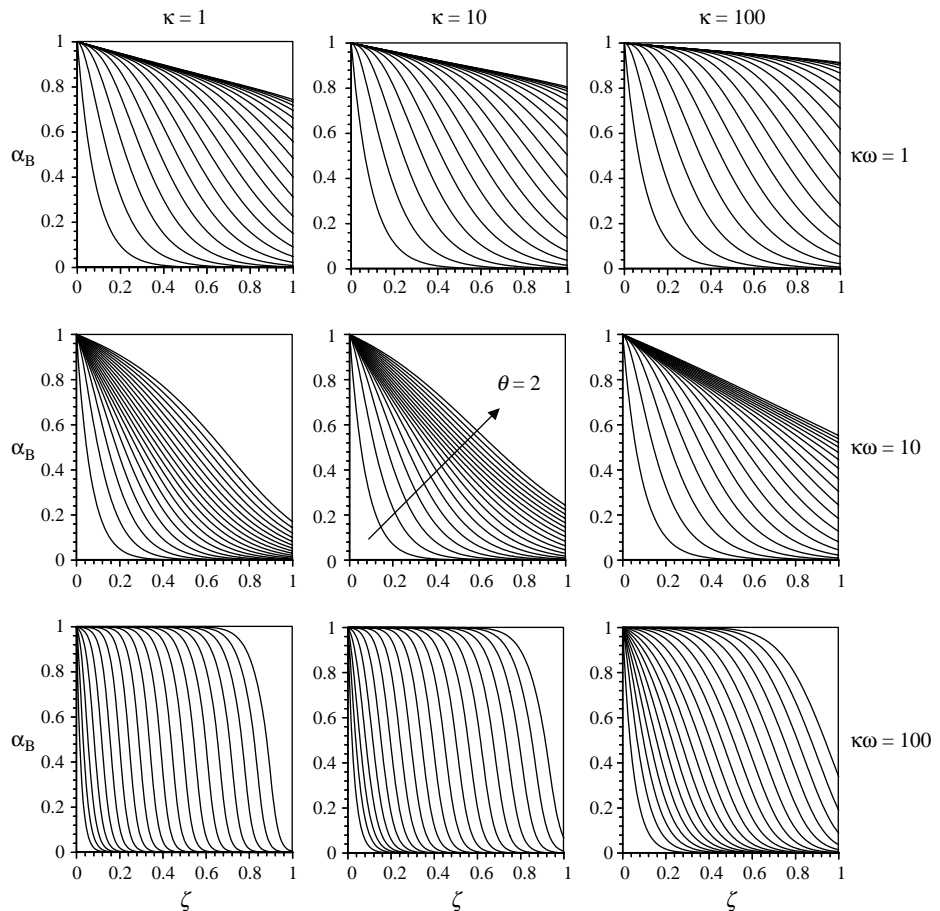

Figure 8.
Fractional extraction in
terms of dimensionless
reaction time for various ϕ

the leaching solvent reaction rate in the particle and has less dependence on the reaction order.

6.2 The results of bed model

The bed model was run for different values of ν, λ, ω and κ and parametric study on numerical results was done. According to the experimentally measured data of heap leaching operations in SCC, the values of ν and λ are about 7 and 0.05, respectively. Thus, having these parameters fixed, there are two variable parameters, ω and κ. The results of bed model for different combinations of $\kappa = 1, 10, 100$ and $\kappa\omega = 1, 10, 100$ are presented.

Changes of leaching solvent concentration in the bed at several time intervals are shown in Figure 9. It is observed that at the first time intervals, leaching solvent concentration in the interior depth of the bed is zero and then the leaching solution flows from the top surface of bed downwards and leaching solvent concentration gradient that is initially steep, approaches zero. It should be mentioned that as the solution moves

Figure 9.
Comparison of acid concentrations in the ore bed for several κ and $\omega (\Delta\theta = 0.1)$

downwards, some leaching solvent is consumed (is diffused into the particles) and therefore the leaching solvent concentration is reduced. When ω is low, after a certain time the leaching solvent concentration throughout the bed becomes invariable and uniform. As a result of this, the reaction is performed uniformly throughout the bed. When ω is increased, i.e. convection resistance becomes higher than the diffusion resistance, the variations of leaching solvent concentration is increased and the zone behavior is observed. It can be seen that when $\kappa\omega = 100$, the variations of leaching solvent concentration occurs in a thin layer of the bed. The leaching solvent concentration at the top and bottom of this layer are 1 and 0, respectively. It is also noticed that when $\kappa\omega = 1$, increasing ω does not result in zone behavior, because in this case the convection resistance is low and the leaching solvent concentration rapidly becomes uniform throughout the bed and therefore the reactions take place uniformly.

The variations of the dissolved mineral concentration in the bed (σ_B) with respect to depth at different time intervals are shown in Figure 10. As the figure shows, σ_B is zero
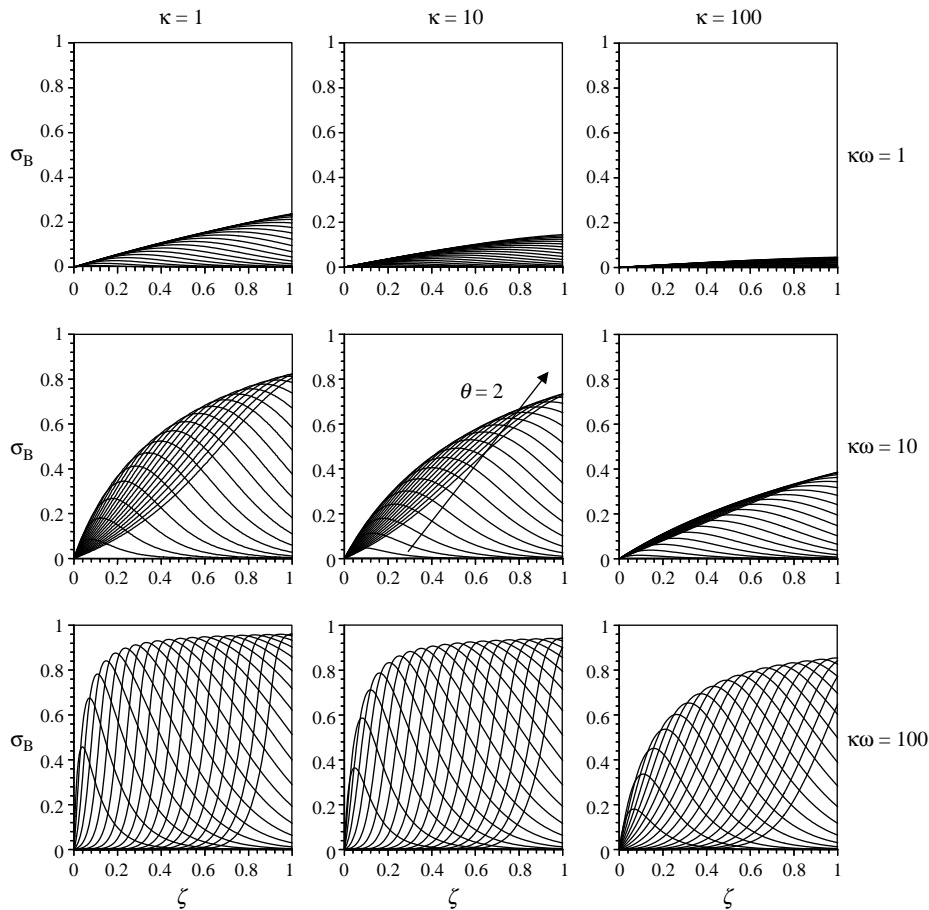
Figure 10.
Comparison of dissolved
mineral concentrations in
the ore bed for several κ
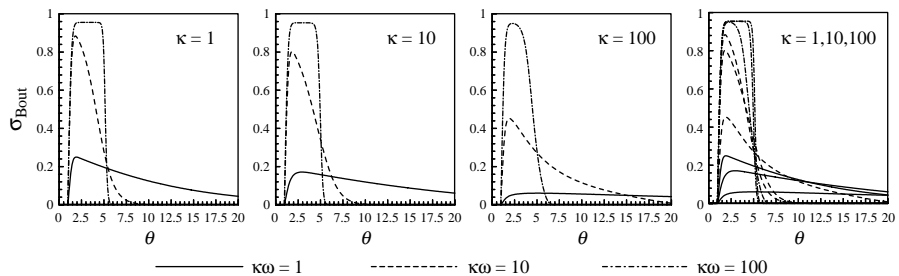and $\omega(\Delta\theta = 0.1)$

at the top and also at the points near the bottom of the bed at the first time intervals. Its peak point happens at a point near the bed surface. As the time goes on, σ_B is increased and the location of the peak point is at a lower location, and finally takes place at the bed exit. The values of σ_B and its peak value are then decreased. At low $\kappa\omega$ values, σ_B is low and decreases as κ is increased or ω is decreased. In other words, the lower the convection resistance in the bed, the higher the solution velocity in the bed. As a result of this, the dissolved mineral is removed more rapidly and will not be accumulated in the bed and therefore σ_B is reduced. When ω is increased, σ_B is also increased. The peak value of σ_B is at a point where leaching solvent is being consumed. At lower points, when leaching solvent concentration is zero, σ_B is also zero, but as the leaching solvent concentration increases with time, σ_B is increased.

Figure 11 shows the variations of dissolved mineral concentration (σ_B) in the solution at the bed exit with respect to time. It can be observed that σ_B is zero when $\theta \leq 1$. At $\theta = 1$ the solution has reached the bed bottom. As the time goes on, σ_{Bout} is increased and approaches a peak value and after that it starts decreasing. The variations of σ_{Bout} becomes more severe when ω is increased, it reaches the peak value very rapidly and then approaches zero. At low ω values, the variations of σ_{Bout} is moderate, so that, at $\kappa = 100$ and $\omega = 0.01$ the value of σ_{Bout} reaches its peak value which is relatively small and will not change considerably with respect to time. The variations of σ_{Bout} for all conditions discussed above are shown in Figure 11.

7. Conclusion
In this paper, the effect of important parameters such as diffusion resistance, convection resistance and chemical reaction resistance have been considered on the concentration distribution of leaching solvent, mineral and dissolved mineral.

It is concluded that at low convection resistance, or high diffusion resistance, the reaction takes place homogeneously throughout the bed and the zone behavior is observed when the convection resistance is increased. It was also noticed that at low concentration resistance or high chemical reaction resistance, the reaction takes place homogeneously throughout the bed. This condition is suitable for heap leaching operations. Therefore, it is recommended to approach the homogeneous reaction condition when preparing ore heaps and designing the relevant operation.

Figure 11.
Comparison of dissolved mineral concentrations in the solution at the bed exit for several κ and $\omega(\Delta\theta = 0.1)$

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